

Dipolar interaction in random-field systems

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1988 J. Phys. A: Math. Gen. 21 L645

(<http://iopscience.iop.org/0305-4470/21/12/005>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 129.252.86.83

The article was downloaded on 01/06/2010 at 05:55

Please note that [terms and conditions apply](#).

LETTER TO THE EDITOR

Dipolar interaction in random-field systems

T Nattermann

Institut für Festkörperforschung der Kernforschungsanlage Jülich, Postfach 1913, D-5170 Jülich, Federal Republic of Germany

Received 21 March 1988

Abstract. It is shown that dipolar interaction breaks the universality of the low-temperature and critical behaviour between the random-field Ising model (RFIM) and the diluted antiferromagnet in an external field (DAFF). For the RFIM, dipolar interaction changes the roughness and the critical exponents; in particular, the upper critical dimensionalities are diminished by one. Dipolar interaction exerts no influence on these exponents in the DAFF, nor on the lower critical dimensionality $d_l = 2$ of both systems.

In many physical systems one encounters quenched random impurities which act as a local symmetry-breaking field on the order parameter. This occurs, for example, in diluted antiferromagnets subjected to a homogeneous external field (DAFF), in monolayers adsorbed on an impure substrate, impure structurally ordered systems (e.g., ferroelectrics), binary mixtures in a porous medium, etc. Most of the macroscopic physical properties related to the disorder are supposed to be described by the random-field Ising model (RFIM) [1], (for a review see [2] and references therein). When only short-range interaction is taken into account, it has been shown that the random field destroys the long-range order of the system in $d \leq d_l = 2$ dimensions. However, in many of the systems under experimental investigation long-range dipolar interaction is expected to play a role. It is the aim of the present letter to elucidate its influence both on the static equilibrium properties of these systems as well as on the domain growth in metastable configurations. It turns out that the dipolar interaction breaks the universality of the RFIM and DAFF.

Following the original ideas of Imry and Ma [1] we consider first the stability of the ferromagnetic up-spin state with respect to the formation of an almost spherical down-spin domain of linear size R in the RFIM. The surface and Zeeman energies of such a configuration are σR^{d-1} and $-MHR^{d/2}$, respectively. The surface tension σ is proportional to the exchange constant J . H denotes the strength of the random field, with $H \ll J$ throughout. The magnetisation M is assumed to be homogeneous everywhere apart from a change of the sign in the domain. Dipolar interaction gives a third contribution W , which results from the difference of the dipolar energy in the domain and in the homogeneous state. In $d = 3$ dimensions, W is given by [3] $W_3 = \frac{1}{2}(2M)^2 NV$ where V denotes the volume and N the demagnetisation factor of the domain ($N = \frac{4}{3}\pi$ for a sphere). In order to generalise W to arbitrary dimensionalities, let us consider three-dimensional dipolar interactions (vanishing as R^{-3}) between dipoles arranged in a d -dimensional space. For an almost spherical domain of radius R , one then obtains $W_d(R) = gM^2 R^{2d-3}$ where R^{2d} results from the integration over dipoles inside and outside of the domain. The shape-dependent coefficient g is

always positive in $d = 3$ whereas for $d = 2$ $g > 0$ ($g < 0$) for \mathbf{M} parallel (perpendicular) to the two-dimensional plane. The total energy of the domain is then

$$E(R) = \sigma R^{d-1} - HMR^{d/2} + gM^2 R^{2d-3}. \quad (1)$$

Here and below we drop all numerical coefficients. For $2 < d$ the Zeeman term increases slower than the exchange and the dipolar term, i.e. $E(R) > E(0)$. For $d < 2$, where domain formation is favoured by the Zeeman term, the dipolar interaction can be neglected because of its fast relative decay on large scales. In $d = 2$ gM^2 just renormalises the bare surface tension $\sigma + gM^2 \rightarrow \tilde{\sigma}$. Additional renormalisation of σ due to random fields leads to a vanishing σ and hence to a stabilisation of the domain state quite analogous to the non-dipolar case [4]. Thus $d_i = 2$ as in the non-dipolar case.

Two complications have not been taken into account so far.

(i) In general domains must not be spherical and the dipolar interaction will favour rotational ellipsoids with the long axis parallel to \mathbf{M} because of the low values of the demagnetisation factor N . Denoting the two principal radii of the ellipsoid by R_1 and R ($R_1 \gg R$), $N \sim (R/R_1)^2 [\ln(2R_1/R) - 1]$ in $d = 3$ dimensions. Keeping R fixed, the domain energy is minimised if

$$R_1 \approx \left(\frac{gM^2}{\sigma} \right)^{1/2} R^{3/2} \left(\ln \frac{2R_1}{R} - 2 \right). \quad (2)$$

Thus (metastable) domains will be preferentially cigar shaped with the long axis parallel to \mathbf{M} . Repetition of the energy estimate (1) for cigar-like domains with $R_1 \approx \gamma^{1/2} R^{d/2}$ ($\gamma \sim gM^2/\sigma$) yields a different expression for $E(R)$ but again $d_i = 2$.

(ii) In order to close the Imry-Ma argument we have to consider a multidomain state. It is now difficult to estimate the energy since the domain pattern will be highly correlated due to the long-range character of the dipolar interaction. Nevertheless, we believe that $E(R)/R^d$, equation (1), gives the correct scale dependence of the energy density. However, it is conceivable that for $d_i \leq 2$ the dipolar interaction leads to a kind of antiferromagnetic ordering of domains quite analogous to those arising in dipolar ferromagnets [5].

Next we consider the Imry-Ma argument for D_{AFF} , which are believed to be good realisations of the R_{FIM} [6]. For a region of size R the dilution leads to an excess of occupied sites on, say, the A lattice, of the order of $R^{d/2}$. Applying an homogeneous external field and neglecting effects from the boundary of this region, the spins on the A lattice will adjust parallel to the field. In a neighbouring region with excess of B -lattice sites, B spins will be parallel to the field. The average magnetisation in both domains is $\mathbf{m}(R) \approx n^{1/2} MR^{-d/2} \mathbf{H}/|H|$ where M , H and n denote the sublattice magnetisation, the external field and the concentration of non-magnetic ions, respectively. Considering the change of the dipolar interaction due to the creation of domains, we have to take into account that the multidomain state has an almost homogeneous magnetisation $\mathbf{m}(R)$ whereas in the corresponding domain-free state $\mathbf{m}(R)$ fluctuates in sign by moving between regions of order R . Thus the multidomain state has a lower dipolar energy given by W_d , but with \mathbf{M} replaced by $\mathbf{m}(R)$ and g by $-cg$ with $c > 0$. Thus

$$\tilde{E}(R) = \sigma R^{d-1} - n^{1/2} HMR^{d/2} - cgnM^2 R^{d-3+x}. \quad (3)$$

From our dimensional analysis of the R dependence of the volume dipolar interaction we get $x = 0$. Contributions from the surface of the domain give $x = d - 2$.

Now, if $d \leq 3$, the dipolar term increases slower than the exchange and random-field terms leaving the Imry–Ma argument unaffected. Thus again $d_i = 2$ as in the non-dipolar case. Despite this result we note that the dipole interaction enters equations (1) and (3) differently: whereas in the RFIM it favours the ordered state here it favours the disordered one (with the exception of the two-dimensional case with \mathbf{M} perpendicular to the layer). Hence we have to expect that there is no longer a universality of the low-temperature phases of the RFIM and the DAFF in the presence of dipolar interaction.

Additional information about the lower critical dimensionality d_l can be obtained from interface calculations [2, 7]. As discussed above, in a dipolar RFIM domain walls will be preferentially parallel to the easy axis of magnetisation x_1 . In a previous paper [8] we have calculated the Hamiltonian for the small distortions $y(x)$ of a flat wall parallel to x_1 ($\mathbf{x} = (x_1, \dots, x_{d-1})$)

$$\mathcal{H}_w = \frac{1}{2} \sum_{\mathbf{q}} \Gamma \mathbf{q}^2 \left(1 + \tilde{\gamma} \frac{q_1^2}{|\mathbf{q}|^d} \right) |y_{\mathbf{q}}|^2 + \int d^{d-1}x \int_0^{y(x)} H(\mathbf{x}, y') dy' \quad (4)$$

where $y_{\mathbf{q}}$ is the Fourier transform of $y(\mathbf{x})$. $\tilde{\gamma} \approx gM^2/\Gamma$ denotes the relative strength of the dipolar interaction and $\Gamma \propto \sigma$ the stiffness of the interface. Interaction with the RF roughens the wall. Repeating the well known Imry–Ma argument [1] for a bump of the wall of height w and of linear extension L_1 parallel to x_1 and L parallel to x_i , $i = 2, \dots, d-1$, we have to minimise the total energy both with respect to w and to L_1 . In this way we get for the width $w(L) = AL^\zeta$, with

$$\zeta = \zeta_{\text{dip}} = \frac{1}{2}(4-d) \quad A = \left(\frac{H}{\Gamma \tilde{\gamma}^{1/2}} \right)^{2/3} \quad L_1 = \tilde{\gamma}^{1/2} L^{d/2}. \quad (5)$$

Thus, typical interface fluctuations are long oval-shaped bumps. Comparing (5) and (2) in three dimensions we see that the form of the bumps reflects the cigar shape of metastable domains. Note that the energy fluctuations $\Delta E \sim L^\chi$ scale with $\chi = 2(\zeta - 1) + (d - 2) + \frac{1}{2}d = \frac{1}{2}d$, contrary to the non-dipolar case where $\chi = \frac{1}{3}(d + 1)$. The roughening exponent $\zeta_{\text{dip}} = \frac{1}{2}(4 - d)$ approaches 1 for $d \rightarrow d_l$. Thus $d_l = 2$ in agreement with our above findings. However, ζ_{dip} is smaller than the non-dipolar result $\zeta = \frac{1}{3}(5 - d)$ [7].

The Hamiltonian (4) was found in [8] by inserting a wall configuration $S_w(\mathbf{x}) = K(x_d - y(\mathbf{x}))$ ($K(x) = 1, 0, -1$ for $x >, =, < 0$) into the Ginzburg–Landau Hamiltonian. Since for a DAFF the dipolar interaction is irrelevant in the limit $\mathbf{q} \approx \mathbf{k}_c$ (see equation (6) below) this should also be the case for the interface Hamiltonian. Hence $\zeta = \frac{1}{3}(5 - d)$, $A = (H/\Gamma)^{2/3}$ and again $d_l = 2$ for the DAFF.

Breakdown of universality is also found from the critical behaviour. In order to be able to perform an expansion around the upper critical dimensionality d_c , I consider now a d -dimensional dipolar interaction in a d -dimensional space. The bare propagator of a φ^4 -field theory corresponding to an Ising ferromagnet with dipolar interaction is $G_0(\mathbf{q}) = (r + \mathbf{q}^2 + \tilde{\gamma} q_1^2 / q^2)^{-1}$. $\tilde{\gamma}$ is again a measure of the relative strength of the dipolar interaction [9]. Adding a RF it is easy to show that the leading contributions in any order of the perturbation theory arise from tree graphs, as in the non-dipolar case [10]. However, because of the $q_1/|q|$ dependence of $G(\mathbf{q})$, which acts like an additional space dimension in all integrals, we get now $d_c = 5$ for the upper critical dimensionality. From the same argument it follows that, to lowest order in ε , the critical exponents of the dipolar RFIM in $d = 5 - \varepsilon$ are identical to those of the RFIM in $d = 6 - \varepsilon$ dimensions (and hence to those of the pure IM in $d = 4 - \varepsilon$). One should expect that in higher

order in ε there is no such simple shift rule because of the complicated direction dependence of the integrands in the perturbation theory.

Next we consider a DAFF for which the Hamiltonian can be written approximately as

$$\begin{aligned} \mathcal{H}_{\text{DAFF}} = & \frac{1}{2} \int_{\mathbf{q}} G_0(\mathbf{q} - \mathbf{k}_c) + \frac{1}{2} u \int_{\mathbf{q}_1} \int_{\mathbf{q}_2} \int_{\mathbf{q}_3} S_{\mathbf{q}_1} S_{\mathbf{q}_2} S_{\mathbf{q}_3} S_{-(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)} \\ & + HS_0 + \int_{\mathbf{q}_1} \int_{\mathbf{q}_2} \delta r(\mathbf{q}_1) S_{-\mathbf{q}_1} S_{-\mathbf{q}_1 - \mathbf{q}_2} \end{aligned} \quad (6)$$

with $\int_{\mathbf{q}} = \int d^d q / (2\pi)^d$. \mathbf{k}_c denotes the wavevector of the antiferromagnetic ordering and $\delta r(\mathbf{q})$ the Fourier transform of the random part of r originating from the dilution

$$\bar{\delta r} = 0 \quad \overline{\delta r(\mathbf{q}) \delta r(\mathbf{q}')} = 2\pi \Delta \delta(\mathbf{q} + \mathbf{q}'). \quad (7)$$

For $H = \delta r = 0$, the system (6) becomes critical at $\mathbf{q} = \mathbf{k}_c$ with Ising critical exponents. Switching on δr (but $H = 0$) new random Ising critical behaviour arises if the specific heat exponent α_0 of the pure model is positive. For $H > 0$ S_0 includes a non-zero part $\langle S_0 \rangle \sim H$ which generates a RF $\langle S_0 \rangle \delta r(\mathbf{q})$. The leading contributions of the perturbation series are again tree graphs with a maximal number of RF insertions. It is easy to show that the random bond insertions, which also exist, are irrelevant in the renormalisation group sense (diagrams which would yield the same divergencies as the maximal divergent RF diagrams, but which contain in addition random bonds, are disconnected before the configuration average is taken and therefore do not exist). Actually, a non-zero $\langle S_0 \rangle$ also produces a S_q^3 term in the effective Hamiltonian, the square of which, however, merely renormalises the S^4 coupling, since one of the S_q has to be non-critical due to momentum conservation. If we include now dipolar interaction there is no change of critical exponents, since expanding the term $\lambda q_z^2 / q^2$ around $\mathbf{q} = \mathbf{k}_c$ does not lead to a critical angular dependence of the propagator for $\mathbf{q} \approx \mathbf{k}$. Thus the critical exponents of the dipolar DAFF are those of the non-dipolar RFIM. In particular, $d_c = 6$.

Experimentally, it is well known that three-dimensional field-cooled RF systems exhibit a domain state at low T , despite the fact that the ordered phase is stable. This behaviour can be understood theoretically by following up the emergence of metastable states when cooling the system through the transition region. Neutron scattering experiments yield a correlation length $R_c(t) \sim H^{-\nu_H} r(t)$. $R_c(t)$ is found by equating the driving force $F_{\text{driv}} = \sigma / R$ acting on the surface of the metastable domain with the pinning force F_{pin} . Pinning arises mainly from the interaction of the rough domain wall with the frozen-in disorder. The maximal pinning force is given by $F_{\text{pin}} \approx \Gamma A^{2/\xi}$ [2, 11]. Thermal hopping makes pinning on small scales ineffective [11, 12]. Repeating the calculation presented in [12] for the dipolar RFIM we get $F_{\text{pin}}(t) = (H^2 / T \tilde{\gamma}^{1/3}) / \ln(1/\tau)$. Interpolating between both expressions for the pinning force, we get

$$R_c(t) \approx \frac{\sigma}{\Gamma} \max \left\{ \left(\frac{\Gamma}{H} \tilde{\gamma}^{1/2} \right)^{8/3(4-d)}, \frac{T \tilde{\gamma}^{1/3}}{H^2} \ln(t/\tau) \right\}. \quad (8)$$

Equation (8) predicts a crossover for ν_H from $\nu_H = \frac{8}{3}/(4-d)$ for short times to $\nu_H = 2$ for long times.

For the DAFF F_{pin} agrees with that for the non-dipolar RFIM [2, 11, 12]. However, the bulk dipolar energy yields a contribution $-gM^2 R^{-3+x}$ to F_{driv} which leads to a slower increase of $R_c(t)$ for small R .

I thank Amnon Aharony for encouragement and a critical reading of the manuscript.

References

- [1] Imry Y and Ma S K 1975 *Phys. Rev. Lett.* **35** 1399
- [2] Nattermann T and Villain J 1988 *Phase Transitions* (New York: Gordon and Breach)
- [3] Döring W 1966 *Handbuch der Physik* vol XVII/2, ed S Flügge (Berlin: Springer) p 428
- [4] Binder K 1983 *Z. Phys.* B **50** 343
- [5] Luttinger J M and Tisza L 1946 *Phys. Rev.* **70** 954
- [6] Fishman S and Aharony A 1979 *J. Phys. C: Solid State Phys.* **12** L729
- [7] Villain J 1982 *J. Physique Lett.* **43** 808
- [8] Nattermann T 1983 *J. Phys. C: Solid State Phys.* **16** 4125
- [9] Larkin A and Kmelnitzkii D E 1969 *Zh. Eksp. Teor. Fiz.* **56** 2087 (1969 *Sov. Phys.-JETP* **29** 1123)
Aharony A and Fisher M E 1973 *Phys. Rev. B* **8** 3323
- [10] Grinstein G 1976 *Phys. Rev. Lett.* **37** 944
Aharony A, Imry Y and Ma S K 1976 *Phys. Rev. Lett.* **37** 1364
- [11] Villain J 1984 *Phys. Rev. Lett.* **52** 1543
Bruinsma R and Aeppli G 1984 *Phys. Rev. Lett.* **52** 1547
- [12] Nattermann T 1985 *Phys. Status Solidi b* **129** 153